Appendix E
Standard Model

E.1 Lagrangian

The Lagrangian of the Standard Model is quite voluminous, and it is already a problem by itself to get all the Feynman rules correct. We will give it a try in this appendix. This section gives the explicit form of the Lagrangian. We assume the simplest Higgs sector. The gauge chosen is the Feynman–'t Hooft gauge. In this gauge the numerator of the vector boson propagators is of the form $\delta_{\mu\nu}$ with respect to the Lorentz indices. There are ghost fields, Higgs ghosts and Faddeev–Popov ghosts. The ghost fields must be included for internal lines, but they should not occur as external lines. They do not correspond to physical particles, but they occur in the diagrams to correct violations of unitarity that would otherwise arise due to the form of the vector boson propagators chosen here. The proof of that fact is really the central part of gauge field theory.

The Lagrangian including the gauge breaking terms can conveniently be subdivided in a number of pieces:

$$\mathcal{L}_{\mathrm{total}} = \mathcal{L}_{\mathrm{c}} + \mathcal{L}_{\mathrm{w}} + \mathcal{L}_{\mathrm{f}} + \mathcal{L}_{\mathrm{fH}} + \mathcal{L}_{\mathrm{fc}} + \mathcal{L}_{\mathrm{FPc}} + \mathcal{L}_{\mathrm{FPw}}\,.$$

These pieces are

- $\mathcal{L}_{\mathrm{c}}$: colour Lagrangian, describing the gluons of quantum chromodynamics and their mutual interactions;
- $\mathcal{L}_{\mathrm{w}}$: weak Lagrangian, describing the vector bosons and their interactions including interactions with the Higgs system;
- $\mathcal{L}_{\mathrm{f}}$: fermion Lagrangian, describing the interactions of the fermions with the weak vector bosons;
- $\mathcal{L}_{\mathrm{fH}}$: fermion–Higgs Lagrangian, describing the interactions of the fermions with the Higgs system;

- $\mathcal{L}_{\rm fc}$: fermion–colour Lagrangian, describing the interactions of the fermions with the gluons;
- $\mathcal{L}_{\rm FPc}$: the Faddeev–Popov ghost Lagrangian of quantum chromodynamics;
- $\mathcal{L}_{\rm FPw}$: the Faddeev–Popov ghost Lagrangian of the weak interactions.

A feature of the present day situation must be mentioned here. In the old days, doing quantum electrodynamics, the important feature was the bare simplicity of the Feynman rules and the small number of particles. The electron and/or muon and the photon were the main players. One vertex, the fermion–photon vertex was all that was needed. As a consequence the number of graphs remained relatively small, in fact usually only one at the one loop level. This fact has made the calculation of as much as four loop effects possible. These very advanced calculations require the use of mechanical procedures, i.e., algebraic computer programs.

To some extent this same simplicity applies to quantum chromodynamics. But in the weak interactions the situation is vastly different. There are now many particles to be considered, and the number of vertices is huge. As a consequence the number of one loop graphs is often already so large that mechanical procedures are needed to process them. These mechanical procedures are very different in nature from those used in quantum electrodynamics. In weak interactions the difficulty is in handling the multitude of vertices and particles.

To establish notation, here are the fields corresponding to the known particles at this moment and the various ghosts.

- A_μ: the photon;
- W^+_μ, W^-_μ, Z^0_μ: the charged and neutral vector bosons of weak interactions;
- ϕ^+, ϕ^-, ϕ^0: the charged and neutral Higgs ghosts;
- H: the physical Higgs particle;
- e^α, ν^α, $\alpha = 1, 2, 3$: three lepton generations;
- d^α_j, u^α_j, $\alpha = 1, 2, 3$, $j = 1, 2, 3$: three quark generations of three colours each.
- Y: the FP ghost associated with the photon;
- X^+, X^- and X^0: the FP ghosts associated with the three vector bosons of weak interactions;

- g^a_μ, $a = 1 \ldots 8$: the eight gluons;
- G^a, $a = 1 \ldots 8$: the eight FP ghosts associated with the gluons.

The FP (= Faddeev–Popov) ghost fields are larger in number than one might think at first. To begin with, remember that they are fictitious particles with fictitious rules. The minus sign for a closed FP loop is just a prescription. Further, the X^- field has nothing to do with the X^+ field, for example it is not true that the X^+ field contains a creation operator for an X^- particle. The way this must be read is this: the X^- field contains the absorption operator for an X^- and the creation operator for an anti-X^-. The anti-X^- field will be denoted by $\bar{X}^-$ and contains the absorption operator for an anti-X^- and the creation operator for an X^-. So, on the one loop level, considering Z^0 self-energy diagrams there will be typically two X-graphs: one with an X^- and one with an X^+ circulating. By contrast, there is only one graph with a circulating charged vector boson. If we had used real vector bosons W^1 and W^2 there would have been two W graphs, and that indeed corresponds more closely to the situation in the FP Lagrangian. In fact, the rule is that corresponding to a real vector field there is a complex FP field. In the case at hand that applies to the FP ghosts associated with the photon and gluon fields. Thus there are $\bar{Y}$ and $\bar{G}^a$ fields.

The parameters occurring in this Lagrangian will be discussed below.

$$
\begin{aligned}
\mathcal{L}_w = \\
& -\partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu \qquad -\tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu \\
& -\tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu \qquad -\tfrac{1}{2}\partial_\mu H \partial_\mu H - \tfrac{1}{2} m_h^2 H^2 \\
& -\partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- \qquad -\tfrac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \tfrac{1}{2c_w^2} M \phi^0 \phi^0 \\
& -\beta_h \left[\tfrac{2M^2}{g^2} + \tfrac{2M}{g} H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-) \right] + \tfrac{2M^4}{g^2}\alpha_h \\
& -igc_w [\partial_\nu Z^0_\mu \left(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu \right) - Z^0_\nu \left(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu \right) \\
& \qquad + Z^0_\mu \left(W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu \right)]
\end{aligned}
$$

(really extension 1 4)

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$\mathcal{L}_W$ ✓

$$-igs_w[\partial_\nu A_\mu \left(W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-\right) - A_\nu \left(W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+\right)$$
$$+A_\mu \left(W_\nu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\nu W_\mu^+\right)]$$
$$-\tfrac{1}{2} g^2 W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- + \tfrac{1}{2} g^2 W_\mu^+ W_\nu^- W_\mu^+ W_\nu^-$$
$$+g^2 c_w^2 \left(Z_\mu^0 W_\mu^+ Z_\nu^0 W_\nu^- - Z_\mu^0 Z_\mu^0 W_\nu^+ W_\nu^-\right)$$
$$+g^2 s_w^2 \left(A_\mu W_\mu^+ A_\nu W_\nu^- - A_\mu A_\mu W_\nu^+ W_\nu^-\right)$$
$$+g^2 s_w c_w [A_\mu Z_\nu^0 (W_\mu^+ W_\nu^- + W_\nu^+ W_\mu^-) - 2 A_\mu Z_\mu^0 W_\nu^+ W_\nu^-]$$
$$-g\alpha_h M [H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-]$$
$$-\tfrac{1}{8} g^2 \alpha_h [H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^-$$
$$+4H^2\phi^+\phi^- + 2(\phi^0)^2 H^2]$$

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$$-gMW_\mu^+ W_\mu^- H - \tfrac{1}{2} g \tfrac{M}{c_w^2} Z_\mu^0 Z_\mu^0 H$$
$$-\tfrac{1}{2} ig [W_\mu^+ (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W_\mu^- (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)]$$
$$+\tfrac{1}{2} g [W_\mu^+ (H\partial_\mu \phi^- - \phi^- \partial_\mu H) + W_\mu^- (H\partial_\mu \phi^+ - \phi^+ \partial_\mu H)]$$
$$+\tfrac{1}{2} g \tfrac{1}{c_w} Z_\mu^0 (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H)$$
$$-ig \tfrac{s_w^2}{c_w} M Z_\mu^0 (W_\mu^+ \phi^- - W_\mu^- \phi^+) \quad +igs_w M A_\mu (W_\mu^+ \phi^- - W_\mu^- \phi^+)$$
$$-ig \tfrac{1-2c_w^2}{2c_w} Z_\mu^0 (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + igs_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+)$$

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$$-\tfrac{1}{4} g^2 W_\mu^+ W_\mu^- [H^2 + (\phi^0)^2 + 2\phi^+\phi^-]$$
$$-\tfrac{1}{8} g^2 \tfrac{1}{c_w^2} Z_\mu^0 Z_\mu^0 [H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+\phi^-]$$
$$-\tfrac{1}{2} g^2 \tfrac{s_w^2}{c_w} Z_\mu^0 \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+) - \tfrac{1}{2} ig^2 \tfrac{s_w^2}{c_w} Z_\mu^0 H (W_\mu^+ \phi^- - W_\mu^- \phi^+)$$
$$+\tfrac{1}{2} g^2 s_w A_\mu \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+)$$
$$+\tfrac{1}{2} ig^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\mu^- \phi^+)$$
$$-g^2 \tfrac{s_w}{c_w} (2c_w^2 - 1) Z_\mu^0 A_\mu \phi^+ \phi^- \quad -g^2 s_w^2 A_\mu A_\mu \phi^+ \phi^- .$$

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The quantities appearing in this Lagrangian are given below. The numerical values are from the Particle Properties Data table 1992. Numbers depend on values assumed for the top quark and

Higgs masses, and there is some ambiguity of interpretation if it comes to including radiative corrections, because to some extent these can be pushed around. The experimentally measured quantities include of course all radiative corrections, and must not be confused with the parameters in the Lagrangian. Consider the numbers given below as indicative, up to the level of, say, 0.5 % .

- The sine and cosine of the weak mixing angle, s_w and c_w. Numerically $s_w^2 = 0.2337$–0.2310.
- The coupling constant g. Its numerical value is linked to the value of the fine-structure constant $\alpha_{\rm em} = 1/137.036$ by means of the relation $\alpha_{\rm w} \equiv g^2/4\pi = \alpha/s_w^2$. This gives $\alpha_{\rm w} = 1/31.8$.
- The vector boson mass $M = 80.22 \pm 0.26$ GeV. Being an unstable particle the definition of its mass is ambiguous on the level of 0.1%.
- Experimentally the neutral vector boson mass $M_0 = 91.173 \pm 0.020$ GeV. That includes of course all radiative corrections. From the point of view of parameters in the Lagrangian, with the simplest Higgs system used here, the neutral vector boson mass M_0 is not a free parameter, and equals the mass of the charged vector boson M divided by c_w: $M_0 = M/c_w$.
- The Higgs mass m_h. Only a lower limit is known: $m_h > 50$ GeV.
- The tadpole constant β_h. It is zero in lowest order, and must be adjusted such that the vacuum expectation value of the Higgs field H remains zero.
- The Higgs scattering parameter α_h. This is not an independent parameter, it is equal to $m_h^2/4M^2$. If this parameter exceeds $\sqrt{1/\alpha_w} \approx 6$ perturbation theory for the Higgs sector breaks down.

The fermion Lagrangian describes the interactions of three lepton and three quark generations. Below, zero neutrino mass is assumed. The generations are labelled using the indices λ and κ, and for example e^2 is the muon, and ν^3 is the τ-neutrino. Likewise u^3 is the top quark. Colour is indexed by means of the index j, taking the values 1–3. The various parameters, including the unitary matrix C (CKM matrix) will be discussed below.

6 $\mathcal{L}_{\mathrm{f}} =$

$$
\begin{aligned}
&-\bar{e}^{\lambda}(\gamma\partial + m_e^{\lambda})e^{\lambda} - \bar{\nu}^{\lambda}\gamma\partial\nu^{\lambda} - \bar{u}_j^{\lambda}(\gamma\partial + m_u^{\lambda})u_j^{\lambda} - \bar{d}_j^{\lambda}(\gamma\partial + m_d^{\lambda})d_j^{\lambda} \\
&+igs_w A_{\mu}[-(\bar{e}^{\lambda}\gamma^{\mu}e^{\lambda}) + \tfrac{2}{3}(\bar{u}_j^{\lambda}\gamma^{\mu}u_j^{\lambda}) - \tfrac{1}{3}(\bar{d}_j^{\lambda}\gamma^{\mu}d_j^{\lambda})] \\
&+\tfrac{ig}{4c_w}Z_{\mu}^0[(\bar{\nu}^{\lambda}\gamma^{\mu}(1+\gamma^5)\nu^{\lambda}) + (\bar{e}^{\lambda}\gamma^{\mu}(4s_w^2 - 1 - \gamma^5)e^{\lambda}) \\
&\qquad +(\bar{d}_j^{\lambda}\gamma^{\mu}(\tfrac{4}{3}s_w^2 - 1 - \gamma^5)d_j^{\lambda}) + (\bar{u}_j^{\lambda}\gamma^{\mu}(1 - \tfrac{8}{3}s_w^2 + \gamma^5)u_j^{\lambda})] \\
&+\tfrac{ig}{2\sqrt{2}}W_{\mu}^{+}[(\bar{\nu}^{\lambda}\gamma^{\mu}(1+\gamma^5)e^{\lambda}) + (\bar{u}_j^{\lambda}\gamma^{\mu}(1+\gamma^5)C_{\lambda\kappa}d_j^{\kappa})] \\
&+\tfrac{ig}{2\sqrt{2}}W_{\mu}^{-}[(\bar{e}^{\lambda}\gamma^{\mu}(1+\gamma^5)\nu^{\lambda}) + (\bar{d}_j^{\kappa}C_{\kappa\lambda}^{\dagger}\gamma^{\mu}(1+\gamma^5)u_j^{\lambda})]
\end{aligned}
$$

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The Fermion–Higgs Lagrangian contains the interactions of the Higgs ghosts with the fermions, and the essentially untested interactions with the Higgs particle. All these interactions are proportional to the ratio of the fermion mass and the vector boson mass.

7. $\mathcal{L}_{\mathrm{fH}} =$

$$
\begin{aligned}
&\tfrac{ig}{2\sqrt{2}}\tfrac{m_e^{\lambda}}{M}[-\phi^{+}(\bar{\nu}^{\lambda}(1-\gamma^5)e^{\lambda}) + \phi^{-}(\bar{e}^{\lambda}(1+\gamma^5)\nu^{\lambda})] \\
&-\tfrac{g}{2}\tfrac{m_e^{\lambda}}{M}[H(\bar{e}^{\lambda}e^{\lambda}) + i\phi^0(\bar{e}^{\lambda}\gamma^5 e^{\lambda})] \\
&+\tfrac{ig}{2M\sqrt{2}}\phi^{+}\left[-m_d^{\kappa}(\bar{u}_j^{\lambda}C_{\lambda\kappa}(1-\gamma^5)d_j^{\kappa}) + m_u^{\lambda}(\bar{u}_j^{\lambda}C_{\lambda\kappa}(1+\gamma^5)d_j^{\kappa})\right] \\
&+\tfrac{ig}{2M\sqrt{2}}\phi^{-}\left[m_d^{\lambda}(\bar{d}_j^{\lambda}C_{\lambda\kappa}^{\dagger}(1+\gamma^5)u_j^{\kappa}) - m_u^{\kappa}(\bar{d}_j^{\lambda}C_{\lambda\kappa}^{\dagger}(1-\gamma^5)u_j^{\kappa})\right] \\
&-\tfrac{g}{2}\tfrac{m_u^{\lambda}}{M}H(\bar{u}_j^{\lambda}u_j^{\lambda}) - \tfrac{g}{2}\tfrac{m_d^{\lambda}}{M}H(\bar{d}_j^{\lambda}d_j^{\lambda}) + \tfrac{ig}{2}\tfrac{m_u^{\lambda}}{M}\phi^0(\bar{u}_j^{\lambda}\gamma^5 u_j^{\lambda}) \\
&\qquad -\tfrac{ig}{2}\tfrac{m_d^{\lambda}}{M}\phi^0(\bar{d}_j^{\lambda}\gamma^5 d_j^{\lambda}).
\end{aligned}
$$

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The various quantities in the fermion Lagrangian are:

- The lepton masses: $m_e = 0.511$ MeV, $m_{\mu} = 105.658$ MeV and $m_{\tau} = 1784$ MeV.
- The quark masses: m_d = 5–15 MeV, m_u = 2–8 MeV, m_s = 100–300 MeV, m_c = 1.3–1.7 GeV, m_b = 4.7–5.3 GeV and $m_t > 91$ GeV.
- The Cabibbo–Kobayashi–Maskawa matrix, specified below.

The most general form of the quark mixing matrix may be transformed into a unitary matrix C, using the symmetries of the Lagrangian. This matrix can be parametrized in various ways; we follow the conventions of the Data booklet. In analysing data, and in the Feynman rules one uses the notation shown here:

$$
C = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}
$$

Experimentally, the ranges for the various parameters are:

$$
C = \begin{bmatrix} 0.9747\text{–}0.9759 & 0.218\text{–}0.224 & 0.002\text{–}0.007 \\ 0.218\text{–}0.224 & 0.9735\text{–}0.9751 & 0.032\text{–}0.054 \\ 0.003\text{–}0.018 & 0.030\text{–}0.054 & 0.9985\text{–}0.9995 \end{bmatrix}
$$

In terms of three angles, θ_{12}, θ_{13} and θ_{23}, all in the first quadrant, and a phase δ, in the range 0–2π, one has:

$$
C = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}
$$

As the reader surely guessed, $s_{12} = \sin\theta_{12}$ etc. The phase angle δ is usually denoted as δ_{13}. The experimental ranges for the angles are at this time:

$$
s_{12} = 0.218\text{–}0.224 \quad s_{23} = 0.032\text{–}0.054 \quad s_{13} = 0.002\text{–}0.007\,.
$$

The quantity δ_{13} is essentially unknown, except that it is non-zero if indeed CP-violation as observed in K-decays is due to this phase. The angle θ_{12} is very close to what used to be the Cabibbo angle, at a time when there was no third generation on the horizon. We repeat that the matrix C is unitary, i.e., $C^{\dagger} = (\tilde{C})^* = C^{-1}$.

The weak Faddeev–Popov ghost Lagrangian is quite complicated, but contains no references to fermions. One must be thankful for little things.

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$$
\begin{aligned}
\mathcal{L}_{\mathrm{FPw}} = \\
& \bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \tfrac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y \\
& + igc_w W_\mu^+(\partial_\mu \bar{X}^0 X^- - \partial_\mu \bar{X}^+ X^0) + igs_w W_\mu^+(\partial_\mu \bar{Y} X^- - \partial_\mu \bar{X}^+ Y) \\
& + igc_w W_\mu^-(\partial_\mu \bar{X}^- X^0 - \partial_\mu \bar{X}^0 X^+) + igs_w W_\mu^-(\partial_\mu \bar{X}^- Y - \partial_\mu \bar{Y} X^+) \\
& + igc_w Z_\mu^0(\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) \\
& \quad + igs_w A_\mu(\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) \\
& - \tfrac{1}{2} gM \left[\bar{X}^+ X^+ H + \bar{X}^- X^- H + \tfrac{1}{c_w^2}\bar{X}^0 X^0 H\right] \\
& + \tfrac{1-2c_w^2}{2c_w} igM \left[\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-\right] \\
& \quad + \tfrac{1}{2c_w} igM \left[\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-\right] \\
& + igMs_w \left[\bar{X}^- Y \phi^- - \bar{X}^+ Y \phi^+\right] + \tfrac{1}{2} igM \left[\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0\right]
\end{aligned}
$$

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Quantum chromodynamics has become quite a domain of specialists. At low energies perturbation theory is not valid, and even at high energies, where perturbative QCD is supposedly valid the situation remains difficult. This is due to colour confinement, the large number of fundamental vector bosons (the gluons), and the fact that they are massless. Anyway, we just cite the Lagrangians for what they are worth. This requires a few preliminaries.

There are eight 3×3 hermitian traceless matrices λ^a, $a = 1 \ldots 8$. They are the straight generalization of the 2×2 Pauli spin matrices, in fact the first three are the Pauli spin matrices in a two dimensional subspace:

$$
\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad
\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad
\lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}
$$

$$
\lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad
\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad
\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}
$$

$$
\lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad
\lambda^8 = \tfrac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}
$$

The commutation and anticommutation rules for these matrices are:

$$
\left[\frac{-i}{2}\lambda^a, \frac{-i}{2}\lambda^b\right] = f^{abc}\,\frac{-i}{2}\lambda^c
$$

$$
\left\{\frac{-i}{2}\lambda^a, \frac{-i}{2}\lambda^b\right\} = -id^{abc}\,\frac{-i}{2}\lambda^c - \frac{1}{3}\delta_{ab}\lambda^0
$$

where λ^0 is the 3×3 unit matrix. The coefficients f are anti-symmetric in all three indices while the d are symmetric in all indices. They are:

f_{123}	f_{147}	f_{156}	f_{246}	f_{257}	f_{345}	f_{367}	f_{458}	f_{678}
1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$
d_{118}	d_{146}	d_{157}	d_{228}	d_{247}	d_{256}	d_{338}	d_{344}	d_{355}
$\frac{1}{\sqrt{3}}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{3}}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{2}$	$\frac{1}{2}$

d_{366}	d_{377}	d_{448}	d_{558}	d_{668}	d_{778}	d_{888}
$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{-1}{2\sqrt{3}}$	$\frac{-1}{2\sqrt{3}}$	$\frac{-1}{2\sqrt{3}}$	$\frac{-1}{2\sqrt{3}}$	$\frac{-1}{\sqrt{3}}$

The structure constants f satisfy a Jacobi identity:

$$
f(g,a,b)f(g,c,d) + f(g,c,a)f(g,b,d) + f(g,b,c)f(g,a,d) = 0\,,
$$

for any a, b, c and d and with g summed over from 1 to 8.
A useful equation for the trace of the product of two λ matrices:

$$
\mathrm{Tr}\left[\lambda^a \lambda^b\right] = 2\delta_{ab}\,.
$$

The colour gluon, the colour fermion and the colour FP Lagrangian are:

$$
\begin{aligned}
1 \quad \mathcal{L}_{\mathrm{c}} &= -\tfrac{1}{2}\partial_\nu g_\mu^a \partial_\nu g_\mu^a \;-\; g_s f^{abc}\partial_\mu g_\nu^a g_\mu^b g_\nu^c \\
&\quad - \tfrac{1}{4} g_s^2\, f^{abc} f^{ade} g_\mu^b g_\nu^c g_\mu^d g_\nu^e \\
2 \quad \mathcal{L}_{\mathrm{fc}} &= \tfrac{1}{2} i g_s (\bar{q}_i^\sigma \gamma^\mu \lambda_{ij}^a q_j^\sigma)\, g_\mu^a \\
3 \quad \mathcal{L}_{\mathrm{FPc}} &= \bar{G}^a \partial^2 G^a \;+\; g_s f^{abc} \partial_\mu \bar{G}^a G^b g_\mu^c
\end{aligned}
$$

In here g_s is the (strong) coupling constant of QCD. The indices a, b and c take the values $1 \ldots 8$ corresponding to the 8 gluons. The lower quark indices i and j take the values $1 \ldots 3$, implying the three colours. The upper index σ, also to be summed over, designates the six quark flavours up, down, strange, charm, bottom and top. All these quarks have identical colour interactions.